Approach to evaluate statistical measures for the thermo-acoustic instability properties of premixed burners

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Abstract

The approach to qualify a flame/burner itself with respect to its thermo-acoustic performance is proposed. Within this approach the thermo-acoustics of a combustion system are divided in two functional elements: 1- the burner described by a known (fixed) TM and 2 – acoustic reflection coefficients on both sided of the burner element. The absolute values and phases of the upstream and downstream reflections are randomly sampled within specified ranges. Next, for each randomly generated configuration of the combustor the system (in)-stability is evaluated. Repeating this test multiple times for the given burner, TM statistical properties of the system instability were calculated. Therefore, the probability that the burner will lead to acoustically unstable combustion if it is installed in an arbitrary (random) combustion chamber can be evaluated. Accordingly, different burners can be compared by their own thermo-acoustic "quality". Furthermore, the role of the burner intrinsic modes (stability and position of the burner TM poles) was revealed. It is shown that for a representative burner Transfer Functions the position of the corresponding TM poles play the role of a point of accumulation of system instability points. This observation suggests a clue for the design of burners with minimal liability to instability.

Introduction

Thermo-acoustic instability of combustion devices is an actual problem hampering and increasing costs of development of almost any combustion appliance. The crux of the instability is related to the coupled nature of the phenomena. Namely, stability or instability of operation of a particular combustion device depends on the acoustic properties of the upstream and downstream parts of the combustor as well as on the thermo-acoustic properties of the burner with certain flame. Therefore, it is difficult to treat the thermo-acoustic performance or to evaluate thermo-acoustic "quality" of some concrete burner when it is taken apart from the concrete combustor (acoustic) environment. Accordingly, it would be advantageous to develop a method to evaluate the thermo-acoustic "figure of merit" of a given burner when it is considered separately from the acoustics of the upstream and downstream ducting parts of the combustor.

Among many different types of burners there is a class of burners for which the thermo-acoustic response of the flame is primarily defined by the oscillation of the acoustic velocity upstream of the burner. A typical representative example of this class is the class of burners dedicated for perfectly premixed combustion [1,2]. Thermo-acoustic characteristics of such burners can be fully described by the so-called Transfer Function (TF) which relates the oscillating part of the combustion heat release rate $Q`(t)$ (flame reaction) to the oscillation of gas velocity $V`(t)$ upstream of the burner (acoustic origin). Within the linear limit, in frequency domain the burner Transfer Function is a complex value function $G(f)$ defined as

$$\boldsymbol{G(f)} = \frac{\boldsymbol{Q`(f)/Q_0}}{\boldsymbol{V`(f)/V_0}}\ . \qquad (1)$$

Here Q_0 and V_0 are corresponding mean (scaling) values.

The flame/burner TF completely characterizes the thermal response of the flame. If to combine this information with hydrodynamic jump conditions across the combustion zone, then the purely acoustic description of the flame/burner behaviour is obtained. Such a description can be conducted for the case of an acoustically thin flame when the streamwise length of the heat release zone (flame) is much smaller than the considered acoustic wavelength. A widely used approach is to describe the burner as an acoustically compact lumped element represented by its acoustic Transfer Matrix (TM). The TM relates a pair of acoustic variables on both sides of given acoustic element. The variables can be pressure $p`$ and velocity $V`$ or, alternately, up- and downstream traveling waves $\mathbf{f}=1/2(V`+p`/\rho_0 c)$ and $\mathbf{g}=1/2(V`-p`/\rho_0 c)$, where $\rho_0 c$ is the specific acoustic impedance of the gas. In the limit of zero mean flow Mach number the link between the TF and TM relating traveling waves is given by the expression [3,4]

$$\boldsymbol{T(s)} = \frac{1}{2}\cdot\begin{pmatrix}\boldsymbol{\epsilon+1+\theta\cdot G(s)} & \boldsymbol{\epsilon-1-\theta\cdot G(s)}\\ \boldsymbol{\epsilon-1-\theta\cdot G(s)} & \boldsymbol{\epsilon+1+\theta\cdot G(s)}\end{pmatrix}. \qquad (2)$$

Here $\varepsilon = {(\rho_{cold}\cdot c_{cold})}/{(\rho_{hot}\cdot c_{hot})}$ is the ratio of specific impedances and $\theta = {T_{hot}}/{T_{cold}} - 1$ is the gas expansion factor across the combustion zone. Because the TM will be used for the analysis of the system stability, the frequency should be considered as a complex quantity $s = \sigma + i\omega$, where the imaginary part is $\omega=2\pi f$ and real part σ indicates either growing/unstable (when $\sigma>0$) or attenuating/stable (when $\sigma<0$) amplitudes.

The acoustic properties of the burner/flame are fully described by its TF or even more completely, by the TM. However, it is absolutely not straightforward to judge what

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the thermo-acoustic quality of the given burner/flame by looking only to its TF or TM elements. It is even not obvious what we mean when we are speaking about the "thermo-acoustic quality" and it should be specified as well.

The ultimate goal of the present contribution is to introduce the notion of thermo-acoustic figure of merit of the burner and to propose a natural interpretation to this intuitive concept. Next, we describe the method how the burner quality can be quantified. This gives the possibility to compare (rank) one burner with respect to others. Furthermore, within the present contribution it is shown that the intrinsic burner properties, i.e. the properties of the burner TM taken separately from the acoustics of adjoint acoustic elements of a combustor, play an important role even for the general case of arbitrary acoustics of the combustor.

Notion of thermos-acoustic quality factor of burner

As it was mentioned above, a direct inspection of the frequency dependency of either gain and phase of the burner TF, or of four elements of the TM does not provide an explicative picture of the thermo-acoustic quality of the given burner. In other words, it is difficult to substantiate an opinion that one burner is better than an other one by just considering their TFs or TMs.

First of all, it is necessary to specify what one means by telling: "thermo-acoustically better". To approach this question it appears to be very fruitful to consider a generic representation of the complete combustor thermo-acoustics as represented on Figure 1.

Figure 1: generic representation of thermo-acoustics of a combustion system.

Here the burner with given flame is the only acoustically active element described by its TM. All passive acoustic elements of the system are collected in two adjoint bloks characterized by their reflection coefficients at the upstream (R_{up}=$\mathbf{f}_1/\mathbf{g}_1$) and downstream ($R_{dn}$=$\mathbf{g}_2/\mathbf{f}_2$) sides of the flame.

In literature two different attempts to approach the problem of the burner element figure of merit are proposed. Within the first concept one evaluates the maximum possible amplification factor for the acoustic energy leaving the burner ($\mathbf{f}_2^2+\mathbf{g}_1^2$) with respect to any combination of incoming acoustic waves $\mathbf{f}_1$ and $\mathbf{g}_2$ [5]. Accordingly, the burner which has the capability to generate a smaller amount of acoustic energy can be treated as the better one. This approach is inherently related to the TM singularity values [4]. The drawbacks of this approach are the following. By "probing" all possible combinations of incoming waves one also includes the combinations, which can be forbidden by the requirement that the upstream and downstream acoustics are passive (i.e. $|R_{up}|<1$ and $|R_{dn}|<1$). A second difficulty is that the maximum energy amplification of one flame/burner can be smaller than of an other one, but the amplification might happen for a wider range of combinations of parameters describing the incoming waves (amplitudes and phases). In this case it is not evident which burner is finally the "better" one.

Within the second approach [6], the theory was developed which allows the following. For a given flame TM one can evaluate requirements (upper bound) for the adjoint acoustics. Particularly, the bound for the absolute values of R_{up} and R_{dn} can be calculated, such that a given burner will be stable, irrespective of the phase of the reflection coefficients. In this way one may characterize the flame stability quality by comparing the requirements of unconditional (in the sense above) stability, which this flame imposes on the adjoint acoustics. The limitation of this approach is that it is only applicable for intrinsically stable flames, i.e. flames with stable poles of its TM (see [4, 7, 8] and discussion below for more details of the notion and nature of the flame intrinsic (in)-stability).

Within the present paper we propose to treat the question of what can be considered as the flame thermo-acoustic quality factor starting from the following intuitive idea. Naturally, one would call burner "A" thermo-acoustically better suited than burner "B" if by placing the burners in different combustion appliances one will encounter instability of combustion less often for burner "A" than for burner "B". Within the schematic representation of Figure 1 this would mean that to test the given flame/burner system one should "probe" many different but acoustically passive combustors, i.e. combinations of R_{up} and R_{dn} and check how many of these combinations will be stable and unstable. Accordingly, the probability of (in)stability, namely the ratio of (un)stable to the total number of tested cases, may serve as natural measure of the burner figure of merit. This definition of a burner thermo-acoustics quality factor seems unambiguous, intuitive and practically relevant.

In the rest of this contribution we develop this idea and show one possible variant of its realization by the calculations of (in)stability probabilities for several examples of artificial but quite representative TFs of flames.

Options to generate passive acoustic terminations

To follow the idea to calculate the (in)stability probability two ingredients are needed: i) the burner/flame TM and ii) a large set of statistically representative acoustic terminations (systems) on the flame up- and downstream sides.

First let's consider the question on how appropriate reflection coefficients can be generated.

At the moment we see three options to generate statistics of R_{up} and R_{dn}. The first one which is the most obvious and simple one for implementation, is just to select absolute values of | R_{up} | and | R_{dn} | as random numbers between 0 and 1. For the phases of R_{up} and R_{dn} one may select random numbers between 0 and 2π. In this case the terminations will be passive but they will be also frequency independent reflection coefficients. What is even more specific is that reflections generated in this way will be constant in the whole s-plane (not only along the $Im(s)=\omega$ axis). All examples presented in this contribution are performed with this simple approach to the statistical sampling of acoustic terminations. We should however remember that it is an artificial situation which is never encountered in practice.

Second, a more realistic method to generate the set of acoustic terminations suitable for the (in)stability probability testing can be the following. The crucial requirement for R_{up} and R_{dn} is that they represent acoustically a passive systems. From the theory of electrical circuits it is known that the entrance impedance of passive system can be described by so-called Positive Real (PR) functions (see for instance [9]). Reversely, any PR function can be realized as the impedance of the system composed of passive elements. Accordingly, if to realize a method to generate randomly PR functions then the corresponding reflection coefficients can be directly calculated and used as the input for the instability tests.

A third option is the following. In practice, some given burner is intended to be used within a certain class of combustion appliances with more or less defined *structure* of upstream and downstream parts of the combustor. The parameters of ducting elements, like tube lengths and diameters, area changes, temperature changes etc. can be varied. In this case one can build generic network models of the upstream and downstream acoustics, which will be specific for the given class of systems. By randomly selecting values of *parameters* of acoustic elements within the range of interest one can build a statistical set of random reflection coefficients which are specific for certain class of appliances and suitable for the instability probability testing.

Selection of flame TF

Our aim at the moment is to demonstrate an application of the instability probability calculation for some examples. Therefore, we have to select and specify the burner/flame thermo-acoustics. As it was mentioned above the acoustic velocity sensitive flames are practically relevant examples, which are extensively studied during the last decades. Typically these are flames/burners dedicated for perfectly premixed combustion. The TF for this class of flames was measured and simulated with CFD for many different configurations. Furthermore, the measured TF was used to calculate the corresponding TM (via Eq. (2)) and further used for the prediction of (in)stability of numerous acoustic configuration. In general, this approach gives reasonably good quality of model prediction, see for instance [10]. Accordingly, by selecting the TF for the tests we will keep in mind typical TF properties of velocity sensitive, premixed flames as a guide.

To recapitulate the common features of the TFs of this class of flames, this will include the following. The gain of the TFs at the limit of low frequency is 1 and the phase is 0 (it is just consequence of energy conservation). At high frequency the gain of the TF decays toward zero. In some frequency range the TF gain may have one or more maxima, sometimes exceeding the value of 1. The phase of the TF in a certain frequency range usually has a close to linear dependency on frequency suggesting a transport time delay feature of the involved physical process.

In literature one can find several theoretical, phenomenological or purely empirical formulas which were proposed to describe a certain set of experimental results. Here, we select the following expression:

$$\boldsymbol{G(s)} = \frac{s_1 \cdot s_1}{(s-s_1)\cdot(s-s_1)} \cdot \boldsymbol{exp}(-s\cdot\tau_0). \qquad (3)$$

This equation describes a second-order system with poles located at s_1 and s_2 and a delay factor with time constant of τ_0. The scaling factor $s_1 s_2$ in the numerator ensures that $|G(\omega\rightarrow 0)|=1$ and that the phase of $G(\omega\rightarrow 0)=0$.

The examples of the TF are presented below. By choosing positions of the TF poles and time delay one can position the TF maximum and easily produce an intrinsically stable or unstable TM (see explanation below). Therefore, this form of TF analytic representation is a convenient working tool for the testing of the instability probability calculation.

Flame settings

To specify acoustically a flame/burner one should provide its TF and input data for ε and θ. Assuming that on both sides of the flame the sound speed and density can be calculated as one for air, namely $\rho(T)=1.29\bullet 273/T$ [kg/m^3] and $c(T)=331\bullet(T/273)^{1/2}$ [m/s] the only required parameters are the temperatures T_{cold} and T_{hot}. In all examples below we specify T_{cold} =300K and T_{hot} =1770K. It results in $\varepsilon=2.43$ and $\theta=4.9$.

The structure of TF in the form of Eq. (3) allows independently changing the TF phase by playing with the value of the time delay τ_0. The TF gain is solely determined by the parameters s_1 and s_2. Changing τ_0 one didn't affect the TF gain, but by changing s_1 and s_2 the phase is affected. This property gives an opportunity to perform a parametric study on the effects of the TF shape on the instability probability. Within this article we will present the results for the flame TF with three different gain dependences on the frequency and for one fixed gain, the value of the time delay (accordingly the TF phase vs frequency) will be varied from zero to several milliseconds.

Calculation procedure

Equations which relate together acoustic waves $\mathbf{f}_1$, $\mathbf{f}_2$, $\mathbf{g}_1$ and $\mathbf{g}_2$ (see Figure 1) are given by

$$\mathbf{f}_1 = R_{up}(s)\cdot\mathbf{g}_1$$
$$\mathbf{f}_2 = T(s)_{1,1}\cdot\mathbf{f}_1 + T(s)_{1,2}\cdot\mathbf{g}_1$$

$$\mathbf{g}_2 = T(s)_{2,1} \cdot \mathbf{f}_1 + T(s)_{2,2} \cdot \mathbf{g}_1$$
$$\mathbf{g}_2 = R_{dn}(s) \cdot \mathbf{f}_2 \, .$$

This homogeneous system of linear equations with respect to variables $\mathbf{f}_1$, $\mathbf{f}_2$, $\mathbf{g}_1$ and $\mathbf{g}_2$ has a nontrivial solution if the determinant of this system is zero. This condition can be explicitly written as

$$\Delta(s) \triangleq |T(s)_{2.2} - R_{dn} \cdot T(s)_{1.2} + R_{up} \cdot T(s)_{2.1} - R_{up}R_{dn} \cdot T(s)_{1.1}| = 0 \, . \quad (4)$$

Complex frequencies s_{eigen} at which this equation is satisfied have an imaginary part ω providing eigen frequency of the oscillations and a real part σ which indicates the growth or attenuation rate of the corresponding eigen mode. If one is interested in unstable modes only, then the solutions with positive σ should be found.

Before we start with the presentation and analysis of the results it is worth to recapitulate the sequence of the calculation steps.

- First we specify the burner/flame thermo-acoustics by selecting parameters for the TF and TM which are s_1, s_2, τ_0, T_{cold}, T_{hot}.
- Next we produce random values for the reflection coefficients R_{up} and R_{dn} using a generator of random numbers with uniform probability distribution between some selected margin values. These values are 0 and 2π for the phases of the reflection coefficients and some values within the range of [0,1] for the absolute value of the reflections. Remember that in this research we assume R_{up} and R_{dn} to be independent of frequency s.
- Next step is to search for solutions of Eq. (4) in the right halve complex plane s. If such solutions exist we record them and repeat the procedure starting from the generation of a new set of reflection coefficients.

In this way one may collect data of unstable eigen frequencies for different combustor configurations with a fixed burner in specific operation point (fixed flame) and use it for further statistical analysis.

Notion of intrinsic thermo-acoustic instability of the flame

In this article we frequently refer to the notion of the flame/burner intrinsic instability. Here, we briefly introduce this concept, for more details see [6-8].

The common point of view on the role of the flame as an active acoustic element is to consider the flame as a dependent amplifier which may increase the amplitude of incoming acoustic waves. The implicit conclusion from this treatment is that if the flame doesn't get any input acoustic signal it doesn't generate an output. Would this conclusion be correct we should expect that the system presented in Figure 1 will be definitely stable if R_{up} and R_{dn} are both zeros. In this case both incoming acoustic waves to the flame ($\mathbf{f}_1$ and $\mathbf{g}_2$) are zero. However, Eq. (4) when both reflection coefficients are zero reduces to a simple relation $T(s)_{2,2}=0$. The corresponding element of the TM is related to the flame TF by (see Eq. (2))

$$T(s)_{2,2} = \epsilon + 1 + \theta \cdot G(s).$$

Accordingly, the flame with anechoic terminations may have eigen frequency which corresponds to a solution of the following equation

$$G(s) = -\frac{\varepsilon+1}{\theta}. \quad (5)$$

A complex solution of this equation s_{intr} determines the mode which depends only on the flame thermo-acoustic properties and therefore it is logical to refer to these modes as to intrinsic flame/burner modes.

Because ε and θ are real and positive numbers, Eq. (5) may have solutions only when the phase of the flame TF equals to $-\pi\pm2\pi N$ (N is an integer number). The solution will have a positive real part (corresponding to the unstable mode) if the gain of the TF exceeds the critical value of $(\varepsilon+1)/\theta$ at the frequency when the phase of the TF crosses π (with period of 2π). For the values of T_{cold} and T_{hot} we have specified above, the critical TF gain value for the instability of intrinsic mode is 0.7.

Results of calculation of instability probability

Now we have all necessary ingredients and notions to present the results. To demonstrate the output of instability probability calculation we will take an example when the TF of the flame is described by Eq. (3) with three different sets of poles s_1 and s_2. The pole positions are selected in such a way that all three TF gains have the same value of 1.1 at the their maxima of their dependences versus frequency. The frequencies at which the TF gains have maximum are are equal to 100, 200 and 300Hz.

First let's consider the case of relatively a big time delay parameter $\tau_0=3ms$. The corresponding TF gains and phases when plotted vs linear frequency (when $\sigma=0$) are presented in Figure 2. One can see that when the TF phases reach for the first time the value of $-\pi$ (at $f\sim120\text{-}140Hz$) all three gains are above the critical value of 0.7 and therefore the intrinsic flame mode, which corresponds to this point will be unstable. The second TF phase crossing of -π (at $f\sim360\text{-}390Hz$) will produce a stable intrinsic mode for the first two TFs with the gain maxima at 100 and 200Hz, but the third flame will have a second intrinsically unstable mode there. At last, intrinsic modes at 680 and 1000Hz are stable for all flames.

This information gives a clue to the interpretation of the data presented in Figure 3 where the result of 5000 tests of random reflection coefficient pairs tested for the system instability are presented.

The absolute values of the used reflections were divided in 5 groups with ranges [0,0.2]; [0.2,0.4]; [0.4,0.6]; [0.6,0.8]

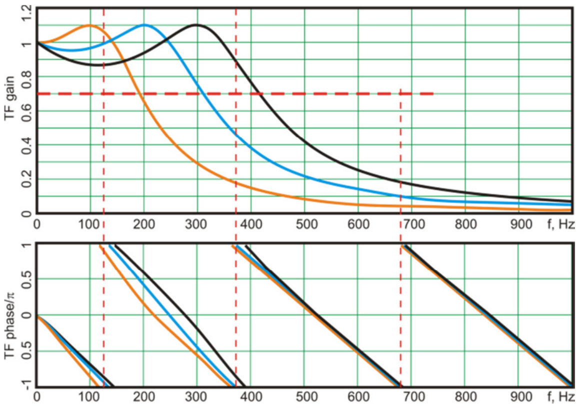

Figure 2: Gain and phase of tested TF versus frequency.

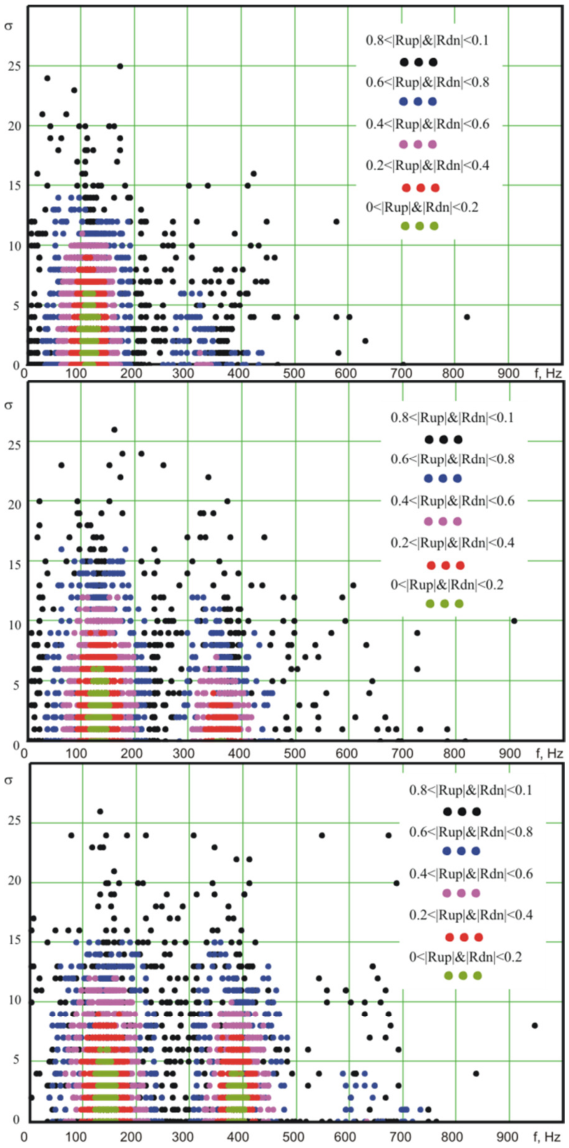

Figure 3: Scatter plots of unstable frequencies for TFs with maxima at 100Hz (up); 200Hz (middle); 300Hz (bottom).

and [0.8,1] represented by the points of different colors plotted in the right halve of the *s*-plane.
Accordingly, each point of these scatters represents one combination of a flame with a random pair of reflection coefficients, which appears to be unstable with the corresponding value of complex eigen frequency.

One can immediately see a strong correlation between the concentration of points and the position of the intrinsic flame modes. When the intrinsic mode is unstable (like one at the frequency of *~120-140Hz*) the points which correspond to small value for the reflection coefficients (green color) are accumulated around the corresponding intrinsically unstable mode of the flame. This is an expected result because by the nature of the flame intrinsic mode it determines the system stability in the case of anechoic terminations. Interestingly, even in the case of a stable intrinsic mode (for instance, one at *f~360Hz*) the concentration of the system unstable modes is higher at the vicinity of the flame intrinsic mode frequency.

A ratio for the number of cases when the system has at least one unstable solution to the total number of tested cases is presented in Figure 4.as function of the reflection coefficients. Since all of the tested TFs have at least one intrinsically unstable flame/burner mode at the limit of small reflections the probability to encounter instability tends to 1. The cumulative (average) probability of instability for these flames is also high as can be judged from Figure 4.

By changing (decreasing) the TF time delay parameter the line of the TF phase vs frequency becomes more flat and this eventually makes all flame intrinsic modes stable.

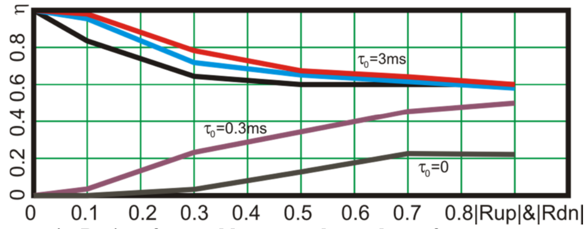

Figure 4: Ratio of unstable to total number of tests vs range of reflection coefficients magnitude

Figure 5 presents the phases behavior as function of frequency of the TFs for the case when the TF gain has maximum at *200Hz* for three time delays $\tau_0=3ms$ (already used above); $\tau_0=0.3ms$ and $\tau_0=0$. At $\tau_0=0.3ms$ the flame intrinsic mode is stable (TF gain is less than *0.7* at *f=~350Hz*). For the case of $\tau_0=0$ there are no crossings of π by the TF phase at all.

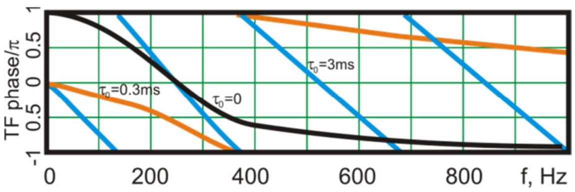

Figure 5: Phase of TF versus frequency for three different time delay factors.

The evaluation of the system instability probability for the cases with smaller time delays is represented by the scatter in Figure 6. Here once again we see the same tendency of accumulation of unstable points around the frequency of the flame intrinsic mode even when this mode is stable. The probability of instability as function of the reflections amplitude (see Figure 4) is zero for the anechoic limit for these flames. The total (average) probability is obviously less for smaller time delay as can be readily seen from Figure 4.

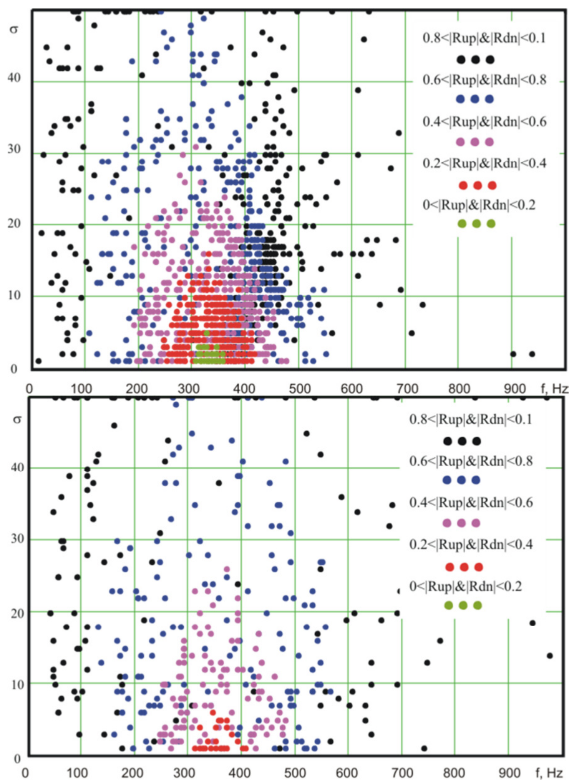

Figure 6: Scatter plots of unstable frequencies for TFs with maxima at 200Hz and time delay of 0.3ms (up) and 0 (bottom).

Conclusions

A new method to characterize the thermo-acoustic quality of a burner/flame represented by its transfer matrix is proposed. The core idea is to rank different burners with respect to each other by evaluating the value of probability of (in)stability. To calculate this probability one should probe a given flame/burner as an acoustic element terminated at the upstream and downstream sides by a set of many different passive terminations with random (or, alternatively, specially selected) reflection coefficients.

One possible implementation of this idea was demonstrated for the case when the reflections were randomly sampled but kept frequency independent. This is an artificial situation, that however allowed to perform fast calculations and facilitate a clear interpretation of results.

The most fundamentally substantial and practically intriguing result of the performed study is the observation of a strong correlation between the frequency range of high probability of instability of a given flame/burner in a combustor and the phenomenon of intrinsic thermo-acoustic behavior of the flame element itself.

From a practical perspective the proposed approach allows qualification (ranking) of different burners by defining its thermo-acoustic figure of merit. Furthermore, the discovered important role of intrinsic modes opens a possibility for dedicated design of burners with improved thermo-acoustic properties.

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